Ultra wide band spatial correlations

Alain SIBILLE (sibille@ensta.fr)
ENSTA, 32 Bd VICTOR, 75739 PARIS cedex 15, France

Abstract

This work addresses the extension of narrow band spatial diversity concepts to ultra wide bands, taking into account the main modulation types which are currently considered, and taking into account candidate combining techniques which could benefit from diversity in a receiver. A series of spatial correlations are thus defined, which are computed analytically in the "extreme UWB regime", and are simulated numerically in the case of a more realistic discrete multipath channel model. According to the receiver architecture, full correlation or full decorrelation can be obtained for moderately close sensors.

Introduction

We investigate here spatial diversity in the context of ultra wide band communications, for which the improvement of the radio link robustness may turn out very useful on account of the stringent power limitation in transmission imposed by existing or pending regulations. We use a one transmitting antenna/several receiving antennas linear system model, where the received signals are correlated with a reference template after passing through antenna and front-end filtering. Such a model is quite general, as the output signals can basically be written as the result of a correlation operation between the received filtered signal and a template : $S(t) = \int F(r(t)).ref(t-t).dt$. According to the choice of the template, a wide selection of basic UWB physical layer schemes can be covered, like pulse amplitude modulation (PAM), binary pulse position modulation (BPPM), direct sequence code division multiplexing access (DSCDMA), switched multiband (SMB), and multiband-OFDM (MBOFDM). Spatial diversity may bring improvement to SIMO or MIMO UWB radio systems [1][2], thus it appears necessary to define "descriptors" helpful to predict the potential improvement expected, and to evaluate the spatial diversity performance of various channels. A series of normalized inter-sensor correlations of the form $r = E(S_1.S_n^*)/E(|S_1|^2)$ are thus defined, where $E(.)$ stands for the expectation operator, and S_1, S_n are output signals for sensors 1 and *n* respectively, upstream of the decision stage :

- r_{SR} (synchronized reference) assumes that optimum synchronisation is achieved on reference sensor 1, resulting in the maximal value of the correlator output for this sensor. For sensor *n*, it is assumed that the clocking times are the same as sensor 1 :
$r_{SR} = E(S_1(t_{\max 1}).S_n^*(t_{\max 1}))/E(|S_1|^2)$ with $S_n(t) = \int r_n(t).ref(t-t).dt$ and $t_{\max 1} = Arg(Max S_1(t))$
- r_{SS} (synchronized sensors) : the clocking times on sensor *n* are now adjusted in order to maximize the correlator output also on this sensor :
$r_{SS} = E(S_1(t_{\max 1}).S_n^*(t_{\max n}))/E(|S_1|^2)$ with $t_1 = Arg(Max S_1(t))$, $t_{\max n} = Arg(Max S_n(t))$
- r_{RD} (Rake decorrelation) assumes perfect (total) Rake combining on reference sensor 1, achieved by filtering with the conjugate transpose of the channel transfer function. Signals from other sensors are subjected to the same filter, which is expected to decorrelate signals very effectively [1] : $F_{RD,1}(r_k(t)) = h^*(\vec{r}_1, -t) \otimes r_k(t)$, where $\otimes$ is the convolution :
$r_{RD} = E(S_1(t_{\max 1}).S_n^*(t_{\max 1}))/E(|S_1|^2)$, $S_n(t) = \int F_{RD,1}(r_n(t)).ref(t-t).dt$, $t_{\max 1} = Arg(Max S_1(t))$

- r_{RC} (Rake combining) assumes total Rake combining on each individual sensor :

$$r_{RC} = E\left(S_1(t_{\max 1}).S_n^*(t_{\max n})\right) / E\left(|S_1|^2\right),\ S_n(t) = \int F_{RD,n}(r_k(t)).ref(t-\tau).dt,\ t_{\max n} = Arg\left(Max S_n(t)\right)$$

- In the case of MBOFDM, the FFT stage output is a vector $\vec{S}_k$ of complex signals for sensor position $\vec{r}_k$. Thus naturally we define the single band correlation as $r_{SBO} = E\left(\vec{S}_1.\vec{S}_n^*\right) / E\left(\left|\vec{S}_1\right|^2\right)$, which is subsequently averaged over all sub-bands.
- Switched multi-bands do not differ from pulsed based modulations, except that the pulse duration is longer. Since the mean frequency is switched at regular times in order to cover the whole bandwidth however, a specific inter-sensor correlation r_{SMB} is defined by averaging the correlation over all sub-bands.

Signal waveforms, channel model and spatial correlations

we assume ideal antennas at both transmitter and receiver, which means here that the antennas are frequency independent when operated in transmission, with no gain (isotropic antennas). In the numerical examples below for single pulse modulations, the transmitted and received signals are shown below, they have been designed in order to respect the FCC mask, with a bandwidth close to 3 GHz.

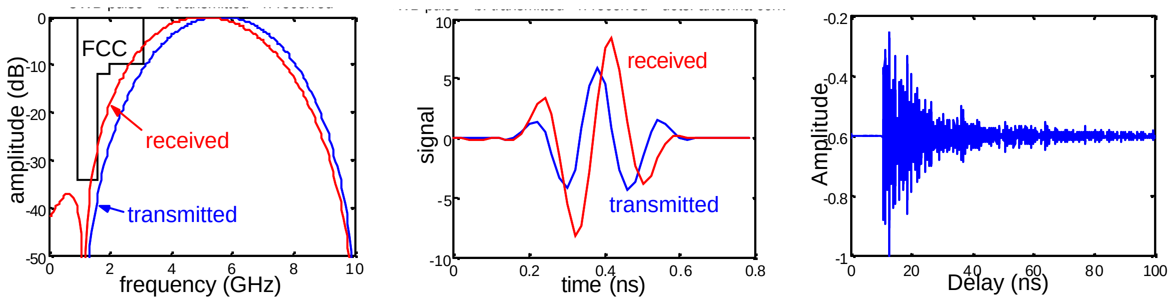

Fig. 1: left : spectra of the transmitted and received signals, in comparison with the FCC mask ; center : temporal waveforms ; right : realization of a simulated channel

We will make use of a space-variant discrete channel model, described as a sum of plane wave multipaths characterized by their delay, their amplitude, and their direction of arrival (DOA). The *real* channel impulse response (CIR) can be written as a function of delay t and receiver antenna position $\vec{r}$ as $h(\vec{r},t) = \sum_i A_i d\left(t - t_i(\vec{r})\right)$, with $t_i\left(\vec{r}\right) = t_{i0} - \vec{r}\cdot\vec{u}_i / c$ ($\vec{u}_i$ is a unitary vector along the DOA at the receiver, c the velocity of light). Path times of arrival (TOA) in each delay bin are distributed according to a Poisson law. Amplitudes are governed by a Ricean distribution, and their sign is randomly chosen (uniform law). The decay of amplitudes with delay is governed by an attenuation exponent. Path DOAs are governed by a gaussian distribution in azimuth, and elevations have systematically been chosen nil. Here we concentrate on a very dense channel, with uniformly distributed DOAs (fig. 1).

With such a model, it is possible to evaluate the various spatial diversity descriptors considered above. Since in a noise-limited receiver with a perfect channel, the optimal template for the correlator is the received signal itself, we choose this ideal received signal for the reference template. Then in the absence of receiver filtering and for a single pulse transmitted at time 0 and received at sensor position $\vec{r}_k$, the received signal writes $rec(\vec{r}_k,t)=\sum_i A_i\, p_r\left(t-t_i\left(\vec{r}_k\right)\right)$. Therefore $S_k(t)=\int\sum_i A_i\, p_r\left(t-t_i\left(\vec{r}_k\right)\right).p_r(t-t).dt$, i.e. $S_k(t)=\sum_i A_i\, \breve{p}_r\left(t-t_i\left(\vec{r}_k\right)\right)$, after defining $\breve{p}_r(t)=\int p_r(t).p_r(t-t).dt$ (pulse auto correlation). Inter-sensor correlations makes an extensive use of the quantity $E\left(S_1\left(t_a\right).S_n^*\left(t_b\right)\right)=E\left(\sum_{i,j} A_i A_j^*\, \breve{p}_r\left(t_a-t_i\left(\vec{r}_1\right)\right).\breve{p}_r^*\left(t_b-t_j\left(\vec{r}_n\right)\right)\right)$. In the "extreme UWB regime, all inter-path delays are well larger than the total pulse autocorrelation duration.

- In the case of r_{SR}, we have $t_a=t_b=t_{iM}\left(\vec{r}_1\right)$, where $i_M=Arg\left(Max\left|A_i\right|\right)$. Then :

$$E\left(S_1\left(t_{\max 1}\right).S_n^*\left(t_{\max 1}\right)\right)=E\left(\sum_{i,j} A_i A_j^*\, \breve{p}_r\left(t_{iM}\left(\vec{r}_1\right)-t_i\left(\vec{r}_1\right)\right).\breve{p}_r^*\left(t_{iM}\left(\vec{r}_1\right)-t_j\left(\vec{r}_n\right)\right)\right)$$

$=E\left(\left|A_{iM}\right|^2 \breve{p}_r(0).\breve{p}_r^*\left(d_{1n}.\cos\left(a_{1n,iM}\right)/c\right)\right)$, where a_{1niM} is the angle between $\left(\vec{r}_n-\vec{r}_1\right)$ and $\vec{u}_{iM}$ and $d_{1n}=\left|\vec{r}_n-\vec{r}_1\right|$. Finally : $r_{SR}=\dfrac{E\left(\left|A_{iM}\right|^2 \breve{p}_r(0).\breve{p}_r^*\left(d_{1n}.\cos\left(a_{1n,iM}\right)/c\right)\right)}{\left|\breve{p}_r(0)\right|^2 E\left(\left|A_{iM}\right|^2\right)}$. Assuming that the angular and amplitude stochastic variables are independently distributed yields $r_{SR}=\dfrac{E\left(\breve{p}_r^*\left(d_{1n}.\cos\left(a_{1n,iM}\right)/c\right)\right)}{\breve{p}_r^*(0)}$. For a single location there is a unique value of $a_{1n,iM}$, thus by taking various values of this angle the synchronized reference inter-sensor correlations depict a series of identical functions except for a dilation factor on the distance axis (fig. 2). If we wish to compute the correlation for a statistical set including several locations but similar path losses, we can approximate the discrete sums by integrals, yielding $r_{SR}=\dfrac{\int P_a(a)\breve{p}_r^*\left(d_{1n}.\cos(a)/c\right).da}{\breve{p}_r^*(0)}$, where P_a is the distribution function of the angles of arrival. For a uniform distribution this writes $r_{SR}=\dfrac{\int \breve{p}_r^*\left(d_{1n}.\cos(a)/c\right).da}{2p.\breve{p}_r^*(0)}$.

- In the case of r_{SS}, we have $t_a=t_{iM}\left(\vec{r}_1\right)$, $t_b=t_{iM}\left(\vec{r}_n\right)$ where $i_M=Arg\left(Max\left|A_i\right|\right)$. Then :

$$E\left(S_1\left(t_{\max 1}\right).S_n^*\left(t_{\max n}\right)\right)=E\left(\sum_{i,j} A_i A_j^*\, \breve{p}_r\left(t_{iM}\left(\vec{r}_1\right)-t_i\left(\vec{r}_1\right)\right).\breve{p}_r^*\left(t_{iM}\left(\vec{r}_n\right)-t_j\left(\vec{r}_n\right)\right)\right)=E\left(\left|A_{iM}\right|^2\right)\left|\breve{p}_r(0)\right|^2$$

resulting in $r_{SS}=1$. Therefore whatever the statistical set the receiver output signals are fully correlated. This can be very simply explained by saying that if the pulses are synchronised on each receiver branch, the output signals are equal.

- In the case of r_{RD}, all received signals are filtered by a common Rake filter, which is that of reference sensor position $\vec{r}_1$: $h^*\left(\vec{r}_1,-t\right)\otimes h\left(\vec{r},t\right)=\sum_{i,j} A_i^*.A_j d\left(t+t_i\left(\vec{r}_1\right)-t_j\left(\vec{r}\right)\right)$. Thus

$$rec(\vec{r}_k,t)=\sum_{i,j} A_i^*.A_j.p_r\left(t+t_i\left(\vec{r}_1\right)-t_j\left(\vec{r}_k\right)\right)$$

$$S_k(t)=\int\sum_{i,j} A_i^*.A_j.p_r\left(t+t_i\left(\vec{r}_1\right)-t_j\left(\vec{r}_k\right)\right).p_r(t-t).dt=\sum_{i,j} A_i^*.A_j.\breve{p}_r\left(t+t_i\left(\vec{r}_1\right)-t_j\left(\vec{r}_k\right)\right)$$

$$E\left(S_1(t_a).S_n^*(t_b)\right)=E\left(\sum_{i,j,k,l} A_i^*.A_j.A_k.A_l^*\breve{p}_r\left(t_a+t_i(\vec{r}_1)-t_j(\vec{r}_1)\right).\breve{p}_r^*\left(t_b+t_k(\vec{r}_1)-t_l(\vec{r}_k)\right)\right)$$

In the case of r_{RD}, $t_a=0$ obviously, since the effect of the time reversed Rake filtering is to combine constructively all multipaths at zero delay. Furthermore in the case of r_{RD} we also have $t_b=0$; $E\left(S_1(0).S_n^*(0)\right)=E\left(\sum_{i,j,k,l} A_i^*.A_j.A_k.A_l^*\breve{p}_r\left(t_i(\vec{r}_1)-t_j(\vec{r}_1)\right).\breve{p}_r^*\left(t_k(\vec{r}_1)-t_l(\vec{r}_n)\right)\right)$

In the extreme UWB regime only terms such that $i=j$ and $k=l$ do not cancel. Finally after some algebra, again assuming a continuous distribution and the independence between amplitude and angular stochastic variables yields :

$r_{RD}=\dfrac{\int |A(a)|^2 P_a(a)\breve{p}_r^*(d_{1n}.\cos(a)/c).da}{\breve{p}_r^*(0)\int |A(a)|^2 P_a(a).da}$. For a Clarke's scenario where $|A(a)|^2 P_a(a)$ does not depend on a we deduce : $r_{RD}=\dfrac{\int \breve{p}_r^*(d_{1n}.\cos(a)/c).da}{2p.\breve{p}_r^*(0)}=r_{SR}$

- In the case of r_{RC}, the Rake filter is adapted for each sensor location, and :

$E\left(S_1(0).S_n^*(0)\right)=E\left(\sum_{i,j,k,l} A_i^*.A_j.A_k.A_l^*\breve{p}_r\left(t_i(\vec{r}_1)-t_j(\vec{r}_1)\right).\breve{p}_r^*\left(t_k(\vec{r}_n)-t_l(\vec{r}_n)\right)\right)$. In the extreme UWB case : $E\left(S_1(0).S_n^*(0)\right)=|\breve{p}_r(0)|^2.E\left(\sum_i |A_i|^2.\sum_k |A_k|^2\right)$, and finally : $r_{RC}=1$.

- In the case of r_{MBO}, we have for a given sensor position $\vec{r}$: $\vec{S}=\sum_i A_i.\exp\left(j\vec{w}t_i(\vec{r})\right)$, where $\vec{w}/2p$ is the vector of frequencies for one of the bands. Thus for two positions $\vec{r}_1$ and $\vec{r}_n$ of the sensors : $E\left(\vec{S}_1.\vec{S}_n^*\right)=E\left(\sum_{i,j} A_i.A_j^*\sum_k \exp\left(jw_k\left(t_{i0}-t_{j0}-(\vec{r}_1.\vec{u}_i-\vec{r}_n.\vec{u}_j)/c\right)\right)\right)$. In the extreme UWB case again we have $i=j$, otherwise the enormous frequency diversity achieved by the ultra wide band will average the exponential to zero : $E\left(\vec{S}_1.\vec{S}_n^*\right)=E\left(\sum_i |A_i|^2\sum_k \exp\left(jw_k\left((\vec{r}_n-\vec{r}_1).\vec{u}_i/c\right)\right)\right)$. With independence between amplitude and angular variables and a continuous approximation, this yields for a single band OFDM :

$r_{SBO}=\dfrac{\sum_k \int P_a(a)\exp\left(jw_k d_{1n}.\cos(a_{1n,k})/c\right).da}{N_F\int P_a(a).da}$, where N_F is the number of frequency tones. In the case of a uniform angular distribution we arrive at $r_{SBO}=\dfrac{1}{N_F}\sum_k r_{NBC}(w_k,d_{1n})$, with $r_{NBC}(w,d)=\dfrac{1}{2p}\int_{-p}^{p}\exp\left(j.wd\cos(a)/c\right).da=J_0(wd/c)$, J_0 being a Bessel function of the first kind. This expression is nothing else than the narrow-band normalized inter-sensor correlation of a Clarkes (omnidirectional) scenario, averaged over all frequency tones. Finally we arrive at r_{MBO} by averaging the correlations over all sub-bands : $r_{MBO}=\sum_{m=1}^{N_B} r_{SBO,m}/N_B$ where N_B is the number of sub-bands.

Results

The various correlations computed above in the extreme UWB regime are shown below, vs. the distance between sensors. It can be seen that for a single location $r_{SR-singleloc}$ exhibits wide oscillations, depending on the DOA of the dominant path. On the other hand r_{SS} and r_{RC} are always equal to 1. When multiple locations cover all angles, we find for r_{SR} a very fast decorrelation. r_{MBO} exhibits a similar fast decorrelation, but in this case it might also obtained for a single location, due to the fact that OFDM involves all multipaths in the output signal. Simulations in the case of the very dense multipath channel shown in fig. 1 (100 random realizations) still qualitatively show a comparable behaviour. For comparison, the normalized narrowband correlation r_{NB} at the center frequency is also shown, as well as the normalized narrowband correlation r_{NBC} for Clarke's scenario at this frequency.

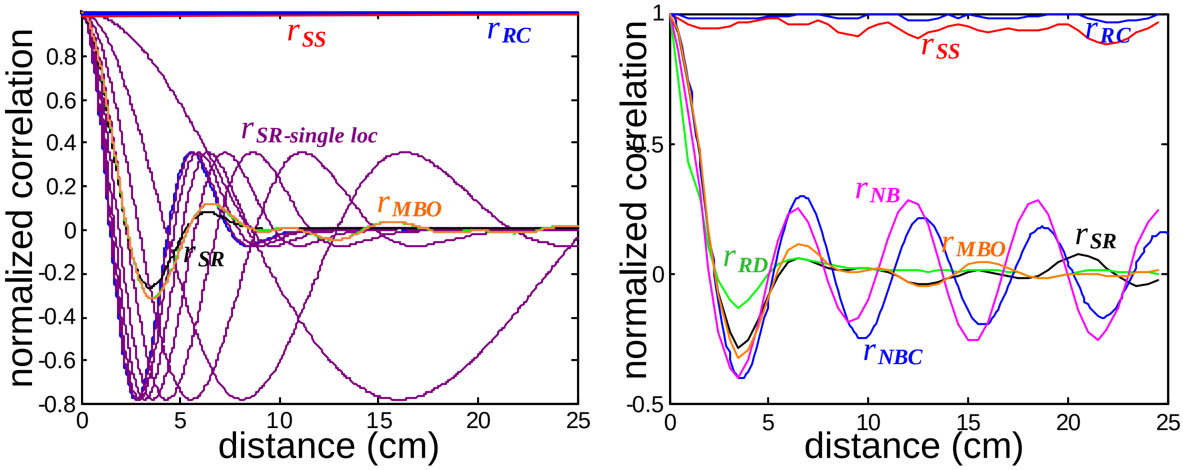

Fig. 2: left : extreme UWB correlations (curves labelled $r_{SR-singleloc}$ cover angles from 10° to 90° by steps of 10°); right : numerical simulations for the dense multipath channel of fig. 1

Acknowledgements

This work has been partly funded by the European Commission within IST project "ULTRAWAVES" (FP5).

References :

[1] "Performance of binary antipodal signaling over the indoor UWB MIMO channel", Martin Weisenhorn and Walter Hirt, International Conference on Communications, Anchorage, May 11-15, 2003

[2] "MIMO diversity for ultra wide band communications", A. Sibille, COST 273 meeting, 15-17 January 2003, Barcelona (Spain), Document TD(03) 071

[3] "On the energy capture of ultra wide bandwidth signals in gaussian noise and dense multipaths", M.Z. Win and R.A. Scholtz, IEEE Commun. Lett. vol. 2, n°9, p. 245 (1998)

[4] "The UWB indoor channel: from statistical model to simulations," D. Cassioli, M. Z. Win and A. F. Molisch, IEEE JSAC Issue Aug. 2002

[5] "Ultra Wideband Indoor Radio Channel Models: Preliminary Results", V. Hovinen, M. Hamalainen and T. Patsi, Proc. IEEE Conf. on UWB Systems and Technol., May 2002

[6] "Measurement results and modeling aspects for the UWB radio channel", K. Kunisch and J. Pamp, COST 273, 19-20 September 2002, Lisbon (Portugal), Document TD(02) 105